

# APPENDICES

## *Globally Optimal Basic Design of Multiple-Unit Heat Exchangers*

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### APPENDIX A

In this appendix, we prove that the correction factors of the individual shells are equal in all three configurations. For the whole exchanger structure with  $N$  units, the following holds:

$$\widehat{LMTD}_T = \frac{(\hat{T}_h^{in} - \hat{T}_c^{out}) - (\hat{T}_h^{out} - \hat{T}_c^{in})}{\ln\left(\frac{\hat{T}_h^{in} - \hat{T}_c^{out}}{\hat{T}_h^{out} - \hat{T}_c^{in}}\right)} = \begin{cases} \frac{(\hat{R}_T - 1)(\hat{T}_c^{out} - \hat{T}_c^{in})}{\ln\left(\frac{1 - \hat{P}_T}{1 - \hat{P}_T \hat{R}_T}\right)} = \frac{(\hat{R}_T - 1)}{\hat{R}_T} \frac{(\hat{T}_h^{in} - \hat{T}_h^{out})}{\ln\left(\frac{1 - \hat{P}_T}{1 - \hat{P}_T \hat{R}_T}\right)} & \hat{R}_T \neq 1 \\ (\hat{T}_h^{in} - \hat{T}_c^{out}) = (\hat{T}_h^{out} - \hat{T}_c^{in}) & \hat{R}_T = 1 \end{cases} \quad (\text{A-1})$$

where

$$\hat{R}_T = \frac{\hat{T}_h^{in} - \hat{T}_h^{out}}{\hat{T}_c^{out} - \hat{T}_c^{in}} = \frac{\hat{m}_c \widehat{Cp}_c}{\hat{m}_h \widehat{Cp}_h} \quad (\text{A-2})$$

$$\hat{P}_T = \frac{\hat{T}_c^{out} - \hat{T}_c^{in}}{\hat{T}_h^{in} - \hat{T}_c^{in}} \quad (\text{A-3})$$

We remind the reader that stream allocation is associated with  $M_h$  and  $M_c$ , which contains information about the number of passes and the corresponding side. Next,

borrowing from previous work (Underwood (1934), Gardner (1941a) and Ivanova and Kanevets (1973), we introduce the following expression for the correction factor:

$$F_{T,al,M_h,M_c}^* = \hat{\alpha}_T^*(\hat{R}_T, \hat{P}_T) f(\hat{R}_T, \hat{P}_T, al, M_h, M_c) \quad (\text{A-4})$$

where

$$\hat{\alpha}_T^*(\hat{R}_T, \hat{P}_T) = \begin{cases} \frac{\ln\left(\frac{1-\hat{P}_T}{1-\hat{P}_T\hat{R}_T}\right)}{(\hat{R}_T-1)} & \hat{R}_T \neq 1 \\ \frac{\hat{P}_T}{1-\hat{P}_T} & \hat{R}_T = 1 \end{cases} \quad (\text{A-5})$$

is known as the number of transfer units for the pure countercurrent case and is independent of the exchanger geometry. In turn,  $f(\hat{R}_T, \hat{P}_T, al, M_h, M_c)$  is a certain function of all four temperatures and the exchanger geometry. Now, from the above equations, we get:

$$F_{T,al,M_h,M_c}^* \widehat{LMTD}_T = \hat{\alpha}_T^* f(\hat{R}_T, \hat{P}_T, al, M_h, M_c) \widehat{LMTD}_T = f(\hat{R}_T, \hat{P}_T, al, M_h, M_c) (\hat{T}_c^{out} - \hat{T}_c^{in}) = f(\hat{R}_T, \hat{P}_T, al, M_h, M_c) \frac{(\hat{T}_h^{in} - \hat{T}_h^{out})}{\hat{R}_T} \quad (\text{A-6})$$

Similar expressions can be written for individual exchangers in each of the structures considered in Figure 1 (provided the correct values of  $R_i^*$  are used).

**Units in Series:** the overall and individual heat balances for  $N$  units are

$$\hat{Q}_T = \hat{m}_h \hat{c}_p \hat{p}_h (\hat{T}_h^{in} - \hat{T}_h^{out}) = \hat{m}_c \hat{c}_p \hat{p}_c (\hat{T}_c^{out} - \hat{T}_c^{in}) \quad (\text{A-7})$$

$$Q_1^S = \hat{m}_h \hat{c}_p \hat{p}_h (\hat{T}_h^{in} - T_h^{\mathcal{S},1}) = \hat{m}_c \hat{c}_p \hat{p}_c (\hat{T}_c^{out} - T_c^{\mathcal{S},2}) \quad (\text{A-8})$$

$$Q_i^S = \hat{m}_h \hat{c}_p \hat{p}_h (T_h^{\mathcal{S},(i-1)} - T_h^{\mathcal{S},i}) = \hat{m}_c \hat{c}_p \hat{p}_c (T_c^{\mathcal{S},i} - T_c^{\mathcal{S},(i+1)}) \quad i = 2, \dots, (N-1) \quad (\text{A-9})$$

$$Q_N^S = \hat{m}_h \hat{c}_p \hat{p}_h (T_h^{\mathcal{S},(N-1)} - \hat{T}_h^{out}) = \hat{m}_c \hat{c}_p \hat{p}_c (T_c^{\mathcal{S},N} - \hat{T}_c^{in}) \quad (\text{A-10})$$

We follow with the area and  $LMTD$  equations for each unit.

$$Q_i^S = U_{i,al,M_h,M_c}^S A_i^S F_{i,al,M_h,M_c}^S LMTD_i^S \quad i = 1, \dots, N \quad (\text{A-11})$$

$$LMTD_1^S = \frac{(\hat{T}_h^{in} - \hat{T}_c^{out}) - (T_h^{\mathcal{S},1} - T_c^{\mathcal{S},2})}{\ln\left(\frac{\hat{T}_h^{in} - \hat{T}_c^{out}}{T_h^{\mathcal{S},1} - T_c^{\mathcal{S},2}}\right)} \quad (\text{A-12})$$

$$LMTD_i^S = \frac{(T_h^{\mathcal{S},(i-1)} - T_c^{\mathcal{S},i}) - (T_h^{\mathcal{S},i} - T_c^{\mathcal{S},(i+1)})}{\ln\left(\frac{T_h^{\mathcal{S},(i-1)} - T_c^{\mathcal{S},i}}{T_h^{\mathcal{S},i} - T_c^{\mathcal{S},(i+1)}}\right)} \quad i = 2, \dots, N-1 \quad (\text{A-13})$$

$$LMTD_N^S = \frac{(T_h^{\mathcal{S},(N-1)} - T_c^{\mathcal{S},N}) - (\hat{T}_h^{out} - \hat{T}_c^{in})}{\ln\left(\frac{T_h^{\mathcal{S},(N-1)} - T_c^{\mathcal{S},N}}{\hat{T}_h^{out} - \hat{T}_c^{in}}\right)} \quad (\text{A-14})$$

As stated, equation (A-6) is also true for each unit in series. Then, using it, we write:

$$F_{1,al,M_h,M_c}^S LMTD_1^S = f(R_1^S, P_1^S, al, M_h, M_c) (\hat{T}_c^{out} - T_c^{S,2}) \quad (A-15)$$

$$F_{i,al,M_h,M_c}^S LMTD_i^S = f(R_i^S, P_i^S, al, M_h, M_c) (T_c^{S,i} - T_c^{S,(i+1)}) \quad (A-16)$$

$$F_{N,al,M_h,M_c}^S LMTD_N^S = f(R_N^S, P_N^S, al, M_h, M_c) (T_c^{S,N} - \hat{T}_c^{in}) \quad (A-17)$$

where we know that  $R_i^S = \hat{R}_N \forall i$ . We now propose the following Lemma:

**Lemma 1:** In a multi-unit heat exchanger arranged in series, with all units featuring equal area and geometry, and with equal and uniform values of physical properties (densities, viscosities, heat capacities, etc.) the corrections factors of each unit ( $F_{i,al,M_h,M_c}^S$ ) are all equal.

$$F_{1,al,M_h,M_c}^S = \dots = F_{i,al,M_h,M_c}^S = \dots = F_{N,al,M_h,M_c}^S \quad (A-18)$$

This lemma was illustrated (not proved) by Nagle (1933) from numerical experimentation, and later also illustrated by Fischer (1938) for one unit with two passes on one side. Based on the observation made by Nagle (1933), Bowman (1936) arrived at the lemma, but only for a certain particular number of passes. Finally, Vengateson (2010) uses the results of this lemma. All three authors derived the lemma by making use of the Underwood expression for  $F_{1,al,1,2}^{Su}$ , and using the same assumptions (same  $U$  and same area for all units). We offer a generalized proof for any number of units and any number of passes, without making any assumptions. Although the expressions were derived thinking of STHes, we remark that they apply to any type of unit, even if they are not stream symmetric, that is, even when the correction factor does depend on fluid allocation.

**Proof:** Using equation (1) and the above equations, we write:

$$U_{i,al,M_h,M_c}^S A_i^S = \frac{\hat{m}_c \widehat{Cp}_c (\hat{T}_c^{out} - T_c^{S,2})}{F_{1,al,M_h,M_c}^S LMTD_1^S} = \dots = \frac{\hat{m}_c \widehat{Cp}_c (T_c^{S,i} - T_c^{S,(i+1)})}{F_{i,al,M_h,M_c}^S LMTD_i^S} = \dots = \frac{\hat{m}_c \widehat{Cp}_c (T_c^{S,N} - \hat{T}_c^{in})}{F_{N,al,M_h,M_c}^S LMTD_N^S} \quad (A-19)$$

where we remind the reader that all  $U_{i,al,M_h,M_c}^S$  are equal and the flowrates and specific heat capacity are the same for all units. Using (A-15) through (A-17) for each unit, and substituting in equation (A-19) we get:

$$\frac{\hat{m}_c \widehat{Cp}_c (\hat{T}_c^{out} - T_c^{S,2})}{f(R_1^S, P_1^S, al, M_h, M_c) (\hat{T}_c^{out} - T_c^{S,2})} = \dots = \frac{\hat{m}_c \widehat{Cp}_c (T_c^{S,i} - T_c^{S,(i+1)})}{f(R_i^S, P_i^S, al, M_h, M_c) (T_c^{S,i} - T_c^{S,(i+1)})} = \dots = \frac{\hat{m}_c \widehat{Cp}_c (T_c^{S,N} - \hat{T}_c^{in})}{f(R_N^S, P_N^S, al, M_h, M_c) (T_c^{S,N} - \hat{T}_c^{in})} \quad (A-20)$$

In the above equations, the temperature differences on the numerator and denominator cancel and realize that  $R_i^S = \hat{R}_T$ , as stated, we conclude that:

$$f(\hat{R}_T, P_1^S, al, M_h, M_c) = \dots = f(\hat{R}_T, P_i^S, al, M_h, M_c) = \dots = f(\hat{R}_T, P_N^S, al, M_h, M_c) \quad (A-21)$$

and consequently, because  $f(\hat{R}_T, P_i^S, al, M_h, M_c)$  is usually invertible, that is, for each value of the function, there is one and only one argument  $P_i^S$ , we conclude that

$$P_1^S = \dots = P_i^S = \dots = P_N^S \quad (A-22)$$

Because  $\alpha_i^S(\hat{R}_N, P_i^S)$  is also invertible, we conclude from (A-5), and (A-22), that

$$\alpha_1^S(\hat{R}_T, P_1^S) = \dots = \alpha_i^S(\hat{R}_T, P_i^S) = \dots = \alpha_N^S(\hat{R}_T, P_N^S) \quad (A-23)$$

Then, using (A-21) and (A-23), we get

$$\alpha_1^S(\hat{R}_T, P_1^S) f(\hat{R}_T, P_1^S, al, M_h, M_c) = \dots = \alpha_i^S(\hat{R}_T, P_i^S) f(\hat{R}_T, P_i^S, al, M_h, M_c) = \dots = \alpha_N^S(\hat{R}_T, P_N^S) f(\hat{R}_T, P_N^S, al, M_h, M_c) \quad (A-24)$$

Next, using (A-4), as applied to any individual exchanger, we obtain

$$F_{1,al,M_h,M_c}^S = \dots = F_{i,al,M_h,M_c}^S = \dots = F_{N,al,M_h,M_c}^S \quad (A-25)$$

**Q.E.D**

**Units in Parallel:** The temperatures for all exchangers are  $\hat{T}_h^{in}$ ,  $\hat{T}_h^{out}$ ,  $\hat{T}_c^{in}$ ,  $\hat{T}_c^{out}$ , then the following expression is trivially straightforward:

$$F_{1,al,M_h,M_c}^P = \dots = F_{i,al,M_h,M_c}^P = \dots = F_{N,al,M_h,M_c}^P \quad (A-26)$$

**Q.E.D**

**Units in Series-Parallel:** Starting from Figure 1c. the heat balances are:

$$\hat{Q}_T = \hat{m}_h \hat{c}_p \hat{p}_h (\hat{T}_h^{in} - \hat{T}_h^{out}) = \hat{m}_c \hat{c}_p \hat{p}_c (\hat{T}_c^{out} - \hat{T}_c^{in}) \quad (\text{A-27})$$

$$Q_1^{SP} = \frac{\hat{m}_c \hat{c}_p \hat{p}_c}{N} (T_c^{SP,1} - \hat{T}_c^{in}) \quad (\text{A-28})$$

$$Q_i^{SP} = \frac{\hat{m}_c \hat{c}_p \hat{p}_c}{N} (T_c^{SP,i} - \hat{T}_c^{in}) \quad i=2, \dots, (N-1) \quad (\text{A-29})$$

$$Q_N^{SP} = \frac{\hat{m}_c \hat{c}_p \hat{p}_c}{N} (T_c^{SP,N} - \hat{T}_c^{in}) \quad (\text{A-30})$$

$$Q_1^{SP} = \hat{m}_h \hat{c}_p \hat{p}_h (\hat{T}_h^{in} - T_h^{SP,1}) \quad (\text{A-31})$$

$$Q_i^{SP} = \hat{m}_h \hat{c}_p \hat{p}_h (T_h^{SP,(i-1)} - T_h^{SP,i}) \quad i=2, \dots, (N-1) \quad (\text{A-32})$$

$$Q_N^{SP} = \hat{m}_h \hat{c}_p \hat{p}_h (T_h^{SP,(N-1)} - \hat{T}_h^{out}) \quad (\text{A-33})$$

We now write:

$$N U_{i,al,M_h,M_c}^{SP} A_i^{SP} = \frac{\hat{m}_c \hat{c}_p \hat{p}_c (T_c^{SP,1} - \hat{T}_c^{in})}{F_{1,al,M_h,M_c}^{SP} LMTD_1^{SP}} = \dots = \frac{\hat{m}_c \hat{c}_p \hat{p}_c (T_c^{SP,i} - \hat{T}_c^{in})}{F_{i,al,M_h,M_c}^{SP} LMTD_i^{SP}} = \dots = \frac{\hat{m}_c \hat{c}_p \hat{p}_c (T_c^{SP,N} - \hat{T}_c^{in})}{F_{N,al,M_h,M_c}^{SP} LMTD_N^{SP}} \quad (\text{A-34})$$

Because equation (A-6) is also true for each unit. We write:

$$F_{i,al,M_h,M_c}^{SP} LMTD_i^{SP} = f(R_i^{SP}, P_i^{SP}, al, M_h, M_c) (T_c^{SP,i} - \hat{T}_c^{in}) \quad (\text{A-35})$$

Substituting in (A-34), and taking into account that  $R_i^{SP} = \hat{R}_T/N$ , we get

$$f\left(\frac{\hat{R}_T}{N}, P_1^{SP}, al, M_h, M_c\right) = \dots = f\left(\frac{\hat{R}_T}{N}, P_i^{SP}, al, M_h, M_c\right) = \dots = f\left(\frac{\hat{R}_T}{N}, P_N^{SP}, al, M_h, M_c\right) \quad (\text{A-36})$$

As in the series case, we conclude that all  $P_i^{SP}$  are equal, and, similarly, all  $\alpha_i^{SP}(\hat{R}_T/N, P_i^{SP})$  are equal, which allows to write that all combined expressions  $\alpha_i^{SP}(\frac{\hat{R}_T}{N}, P_i^{SP}) f(\frac{\hat{R}_T}{N}, P_i^{SP}, al, M_h, M_c)$  are equal. Thus,

$$F_{1,al,M_h,M_c}^{SP} = \dots = F_{i,al,M_h,M_c}^{SP} = \dots = F_{N,al,M_h,M_c}^{SP} \quad (\text{A-37})$$

The proof for the other series parallel option (**PS**) (Figure 1d) has the same derivation:

$$U_{i,al,M_h,M_c}^{SP} A_i^{SP} = \frac{\hat{m}_h \hat{c}_p \hat{p}_h (\hat{T}_h^{in} - T_h^{SP,1})}{F_{1,al,M_h,M_c}^{SP} LMTD_1^{SP}} = \dots = \frac{\hat{m}_h \hat{c}_p \hat{p}_h (\hat{T}_h^{in} - T_h^{SP,i})}{F_{i,al,M_h,M_c}^{SP} LMTD_i^{SP}} = \dots = \frac{\hat{m}_h \hat{c}_p \hat{p}_h (\hat{T}_h^{in} - T_h^{SP,N})}{F_{N,al,M_h,M_c}^{SP} LMTD_N^{SP}} \quad (\text{A-38})$$

Using equation (A-6), we write:

$$F_{i,al,M_h,M_c}^{SP} LMTD_i^{SP} = f(R_i^{SP}, P_i^{SP}, al, M_h, M_c) \frac{(\hat{T}_h^{in} - T_h^{SP,i})}{R_i^{SP}} \quad (\text{A-39})$$

which after substitution in (A-38) leads to (A-36), and (A-37).

## APPENDIX B

We derive the value of the correction factor for the whole set of units.

**Units in Series:** We start writing the *LMTD* expressions

$$\hat{Q}_T = \begin{cases} U_{1,al,M_h,M_c}^S A_T^S \hat{F}_{T,al,M_h,M_c}^S \frac{(\hat{R}_T-1)(\hat{T}_c^{out}-\hat{T}_c^{in})}{\ln\left(\frac{1-\hat{P}_T}{1-\hat{P}_T\hat{R}_T}\right)} & \hat{R}_T \neq 1 \\ U_{1,al,M_h,M_c}^S A_T^S \hat{F}_{T,al,M_h,M_c}^S (\hat{T}_h^{in} - \hat{T}_c^{out}) & \hat{R}_T = 1 \end{cases} \quad (B-1)$$

$$Q_1^S = \begin{cases} U_{1,al,M_h,M_c}^S A_1^S F_{1,al,M_h,M_c}^S \frac{(\hat{R}_T-1)(T_c^{out}-T_c^{S,2})}{\ln\left(\frac{1-P_1^S}{1-P_1^S\hat{R}_T}\right)} & \hat{R}_T \neq 1 \\ U_{1,al,M_h,M_c}^S A_1^S F_{1,al,M_h,M_c}^S (T_h^{S,2} - \hat{T}_c^{out}) & \hat{R}_T = 1 \end{cases} \quad (B-2)$$

$$Q_i^S = \begin{cases} U_{i,al,M_h,M_c}^S A_i^S F_{i,al,M_h,M_c}^S \frac{(\hat{R}_T-1)(T_c^{S,i}-T_c^{S,(i+1)})}{\ln\left(\frac{1-P_i^S}{1-P_i^S\hat{R}_T}\right)} & \hat{R}_T \neq 1 \\ U_{i,al,M_h,M_c}^S A_i^S F_{i,al,M_h,M_c}^S (T_h^{S,(i+1)} - T_c^{S,i}) & \hat{R}_T = 1 \end{cases} \quad (B-3)$$

$$Q_N^S = \begin{cases} U_{N,al,M_h,M_c}^S A_N^S F_{N,al,M_h,M_c}^S \frac{(\hat{R}_T-1)(T_c^{S,N}-\hat{T}_c^{in})}{\ln\left(\frac{1-P_N^S}{1-P_N^S\hat{R}_T}\right)} & \hat{R}_T \neq 1 \\ U_{N,al,M_h,M_c}^S A_N^S F_{N,al,M_h,M_c}^S (\hat{T}_h^{in} - T_c^{S,N}) & \hat{R}_T = 1 \end{cases} \quad (B-4)$$

Adding (B-3), (B-4) and (B-5) for all units, using the fact that all areas, heat transfer coefficients, and correction factors are the same (Lemma 1, above), we get

$$\begin{aligned} \hat{Q}_T = \sum_{i=1..N} Q_i^S &= U_{1,al,M_h,M_c}^S \frac{A_T^S}{N} F_{1,al,M_h,M_c}^S \begin{cases} \frac{(\hat{R}_T-1)}{\ln\left(\frac{1-P_1^S}{1-P_1^S\hat{R}_T}\right)} \sum_i (T_c^{S,i} - T_c^{S,(i+1)}) & \hat{R}_T \neq 1 \\ \sum_i (T_h^{S,(i+1)} - T_c^{S,i}) & \hat{R}_T = 1 \end{cases} = \\ &= U_{1,al,M_h,M_c}^S \frac{A_T^S}{N} F_{1,al,M_h,M_c}^S \begin{cases} \frac{(\hat{R}_T-1)}{\ln\left(\frac{1-P_1^S}{1-P_1^S\hat{R}_T}\right)} (\hat{T}_c^{out} - \hat{T}_c^{in}) & \hat{R}_T \neq 1 \\ N(\hat{T}_h^{in} - \hat{T}_c^{out}) & \hat{R}_T = 1 \end{cases} \end{aligned} \quad (B-5)$$

We now write:

$$\prod_i \left( \frac{1-P_i^S}{1-P_i^S\hat{R}_T} \right) = \left( \frac{1-P_1^S}{1-P_1^S\hat{R}_T} \right)^N = \frac{(\hat{T}_h^{in}-\hat{T}_c^{out})}{(T_h^{S,1}-T_c^{S,2})} \dots \frac{(T_h^{S,(i-1)}-T_c^{S,i})}{(T_h^{S,i}-T_c^{S,(i+1)})} \dots \frac{(T_h^{S,(N-1)}-T_c^{S,N})}{(\hat{T}_h^{out}-\hat{T}_c^{in})} = \frac{\hat{T}_h^{in}-\hat{T}_c^{out}}{\hat{T}_h^{out}-\hat{T}_c^{in}} = \frac{1-\hat{P}_T}{1-\hat{P}_T\hat{R}_T} \quad \hat{R}_T \neq 1 \quad (B-6)$$

Therefore

$$N \ln \left( \frac{1-P_1^S}{1-P_1^S\hat{R}_T} \right) = \ln \left( \frac{1-\hat{P}_T}{1-\hat{P}_T\hat{R}_T} \right) \quad \hat{R}_T \neq 1 \quad (B-7)$$

which renders:

$$P_1^S = \frac{1 - \left(\frac{1 - \hat{P}_T}{1 - \hat{P}_T \hat{R}_T}\right)^{1/N}}{\hat{R}_T - \left(\frac{1 - \hat{P}_T}{1 - \hat{P}_T \hat{R}_T}\right)^{1/N}} \quad \hat{R}_T \neq 1 \quad (\text{B-8})$$

Taking the limit of this expression for  $\hat{R}_T \rightarrow 1$  (using L'Hôpital rule), one obtains

$$P_1^S = \frac{\hat{P}_T}{N + (1 - N)\hat{P}_T} \quad \hat{R}_T = 1 \quad (\text{B-9})$$

Then, substituting and comparing with (B-1), and because all correction factors are equal, we get (for any value of  $\hat{R}_T$ ):

$$F_{T,al,M_h,M_c}^S = F_{i,al,M_h,M_c}^S \quad (\text{B-10})$$

**Remark:** the above result only applies to arrangements where the flow arrangement is equal in all units or where slightly different flow arrangements still render the same result. For example, for multiple-pass STHs, the flow arrangement can be co-current or counter-current (Figure S-4 in Supplemental Material). This gives rise to 4 possible arrangements for two interconnected shells with 1 shell pass and 2 tube passes, of which the co-current/counter-current and the counter-current/co-current are the most popular. Nagle (1933) was the first to point out that the correction factor curves are the same “whether the shell-side fluid enters near the fixed head or near the floating head”. Later Underwood (1934) analyzed the differential equations corresponding to both cases. Finally, Bowman (1940) states that “The correction factor is the same whether the shell-side fluid enters at the fixed or the floating head”. We integrated the differential equations in Matlab and obtained the same outlet temperatures irrespective of the orientation.

**Units in Parallel:** In this case, using (A-1)

$$Q_i^P = U_{i,al,M_h,M_c}^P \frac{A_T^P}{N} F_{i,al,M_h,M_c}^P \begin{cases} \frac{(\hat{R}_T - 1)(\hat{T}_c^{out} - \hat{T}_c^{in})}{\ln\left(\frac{1 - \hat{P}_T}{1 - \hat{P}_T \hat{R}_T}\right)} & \hat{R}_T \neq 1 \\ (\hat{T}_h^{out} - \hat{T}_c^{in}) & \hat{R}_T = 1 \end{cases} \quad (\text{B-11})$$

where the inlet and outlet temperatures are the same (the split is in equal proportions). Then, given (A-25) and the fact that  $R_i^{\mathcal{P}} = \hat{R}_T$  as well as  $P_i^{\mathcal{P}} = \hat{P}_T$ , we have

we obtain

$$F_{T,al,M_h,M_c}^{\mathcal{P}} = F_{i,al,M_h,M_c}^{\mathcal{P}} \quad (\text{B-12})$$

which shows that the overall correction factor is equal to the correction factor for each unit.

**Units in Series-Parallel:** Consider the case of Figure 1c. To obtain the correction factor corresponding to the whole structure ( $F_{T,M_s,M_t}^{\mathcal{SP}}$ ), we start with the heat balances for  $R_i^{\mathcal{SP}} \neq 1$

$$Q_1^{\mathcal{SP}} = U_{1,al,M_h,M_c}^{\mathcal{SP}} A_1^{\mathcal{SP}} F_{1,M_{s_1},M_{s_2}}^{\mathcal{SP}} \frac{(R_1^{\mathcal{SP}} - 1)(T_c^{\mathcal{SP},1} - \hat{T}_c^{in})}{\ln\left(\frac{1 - P_1^{\mathcal{SP}}}{1 - P_1^{\mathcal{SP}} R_1^{\mathcal{SP}}}\right)} \quad R_1^{\mathcal{SP}} \neq 1 \quad (\text{B-13})$$

$$Q_i^{\mathcal{SP}} = U_{i,al,M_h,M_c}^{\mathcal{SP}} A_i^{\mathcal{SP}} F_{i,al,M_h,M_c}^{\mathcal{SP}} \frac{(R_i^{\mathcal{SP}} - 1)(T_c^{\mathcal{SP},i} - \hat{T}_c^{in})}{\ln\left(\frac{1 - P_i^{\mathcal{SP}}}{1 - P_i^{\mathcal{SP}} R_i^{\mathcal{SP}}}\right)} \quad R_i^{\mathcal{SP}} \neq 1 \quad (\text{B-14})$$

$$Q_N^{\mathcal{SP}} = U_{N,al,M_h,M_c}^{\mathcal{SP}} A_N^{\mathcal{SP}} F_{N,al,M_h,M_c}^{\mathcal{SP}} \frac{(R_N^{\mathcal{SP}} - 1)(T_c^{\mathcal{SP},N} - \hat{T}_c^{in})}{\ln\left(\frac{1 - P_N^{\mathcal{SP}}}{1 - P_N^{\mathcal{SP}} R_N^{\mathcal{SP}}}\right)} \quad R_N^{\mathcal{SP}} \neq 1 \quad (\text{B-15})$$

Then, adding (B-11), (B-12) and (B-13) for all units and using  $A_i^{\mathcal{SP}} = A_T^{\mathcal{SP}}/N$  together with  $R_i^{\mathcal{SP}} = \hat{R}_T/N$  as well as (A-37) we get

$$\begin{aligned} \hat{Q}_T &= \sum_{i=1,\dots,N} Q_i^{\mathcal{SP}} = U_{1,al,M_h,M_c}^{\mathcal{SP}} \frac{A_T^{\mathcal{SP}}}{N} F_{1,M_h,M_c}^{\mathcal{SP}} \frac{(\hat{R}_T/N - 1) \sum_{i=1,\dots,N} (T_c^{\mathcal{SP},i} - \hat{T}_c^{in})}{\ln\left(\frac{1 - P_1^{\mathcal{SP}}}{1 - P_1^{\mathcal{SP}} R_1^{\mathcal{SP}}}\right)} = \\ &U_{1,al,M_h,M_c}^{\mathcal{SP}} A_T^{\mathcal{SP}} F_{1,M_h,M_c}^{\mathcal{SP}} \frac{(\hat{R}_T/N - 1)(\hat{T}_c^{out} - \hat{T}_c^{in})}{\ln\left(\frac{1 - P_1^{\mathcal{SP}}}{1 - P_1^{\mathcal{SP}} R_1^{\mathcal{SP}}}\right)} \quad \hat{R}_T/N \neq 1 \quad (\text{B-16}) \end{aligned}$$

where we used the fact that  $\sum_{i=1,\dots,N} T_c^{\mathcal{SP},i} = N \hat{T}_c^{out}$ , which is the energy balance at the mixer.

As already established in Appendix A,  $P_i^{\mathcal{SP},i}$ ,  $\alpha_i^{\mathcal{SP}}(\hat{R}_T/N, P_i^{\mathcal{SP}})$  and  $F_{i,al,M_h,M_c}^{\mathcal{SP}}$  are equal for all exchangers. First, note that.

$$\begin{aligned} \prod_i (1 - P_i^{\mathcal{SP}} R_i^{\mathcal{SP}}) &= (1 - P_1^{\mathcal{SP}} R_1^{\mathcal{SP}})^N = \frac{(T_h^{\mathcal{SP},1} - \hat{T}_c^{in})}{\hat{T}_h^{in} - \hat{T}_c^{in}} \dots \frac{T_h^{\mathcal{SP},i} - \hat{T}_c^{in}}{T_h^{\mathcal{SP},(i-1)} - \hat{T}_c^{in}} \dots \frac{\hat{T}_h^{out} - \hat{T}_c^{in}}{T_h^{\mathcal{SP},(N-1)} - \hat{T}_c^{in}} = \\ &\frac{\hat{T}_h^{out} - \hat{T}_c^{in}}{\hat{T}_h^{in} - \hat{T}_c^{in}} = 1 - \hat{P}_T \hat{R}_T \quad (\text{B-17}) \end{aligned}$$

Therefore, because all  $P_i^{\mathcal{SP}}$  are equal (Appendix A), we have

$$(1 - P_i^{\mathcal{SP}} \hat{R}_T/N) = (1 - \hat{P}_T \hat{R}_T)^{1/N} \quad (\text{B-18})$$

From this expression, one can also write:

$$P_i^{SP} = \frac{1-(1-\hat{P}_T \hat{R}_T)^{1/N}}{\hat{R}_T/N} \quad (\text{B-19})$$

Therefore:

$$\hat{Q}_T = U_{1,al,M_h,M_c}^{SP} A_T^{SP} F_{1,M_{S_1},M_{S_2}}^{SP} \frac{(\hat{R}_T/N-1)(\hat{T}_c^{out}-\hat{T}_c^{in})}{\ln\left(\frac{1-\frac{N}{\hat{R}_T}[1-(1-\hat{P}_T \hat{R}_T)^{1/N}]}{(1-\hat{P}_T \hat{R}_T)^{1/N}}\right)} \hat{R}_T/N \neq 1 \quad (\text{B-20})$$

which, when compared with the overall expression:

$$\hat{Q}_T = U_{1,al,M_h,M_c}^{SP} A_T^{SP} F_{T,al,M_h,M_c}^{SP} \frac{(\hat{R}_T-1)(\hat{T}_c^{out}-\hat{T}_c^{in})}{\ln\left(\frac{1-\hat{P}_T}{(1-\hat{P}_T \hat{R}_T)}\right)} \hat{R}_T \neq 1 \quad (\text{B-21})$$

renders

$$F_{T,al,M_h,M_c}^{SP} = F_{i,al,M_h,M_c}^{SP} \frac{\left(\frac{\hat{R}_T-1}{N}\right)}{(\hat{R}_T-1)} \frac{\ln\left(\frac{1-\hat{P}_T}{(1-\hat{P}_T \hat{R}_T)}\right)}{\ln\left(\frac{1-\frac{N}{\hat{R}_T}[1-(1-\hat{P}_T \hat{R}_T)^{1/N}]}{(1-\hat{P}_T \hat{R}_T)^{1/N}}\right)} \hat{R}_T/N \neq 1, \hat{R}_T \neq 1 \quad (\text{B-22})$$

Taking the limit of the expression for  $\hat{R}_T \neq 1$  for  $\hat{R}_T \rightarrow 1$  (using L'Hôpital rule), one obtains

$$F_{T,al,M_h,M_c}^{SP} = F_{1,M_h,M_c}^{SP} \frac{(N-1)\left(\frac{\hat{P}_T}{(1-\hat{P}_T)}\right)}{N \ln\left(\frac{1}{(1-\hat{P}_T)^{\frac{1}{N}}}\right)} \hat{R}_T = 1 \quad (\text{B-23})$$

Finally, we take the limit of the expression for  $\hat{R}_T/N \neq 1$  for  $\hat{R}_T \rightarrow N$  (using L'Hôpital rule), to obtain

$$F_{T,al,M_h,M_c}^{SP} = F_{i,al,M_h,M_c}^{SP} \frac{\ln\left(\frac{1-\hat{P}_T}{(1-\hat{P}_T N)}\right)}{(N-1) \left\{ \frac{1}{1-(1-\hat{P}_T N)^{\frac{1}{N}}} \right\}} \frac{\hat{R}_T}{N} = 1 \quad (\text{B-24})$$

The derivation for the arrangement in Figure 1d is similar: For  $R_i^{PS} \neq 1$

$$Q_1^{PS} = U_{1,al,M_h,M_c}^{PS} A_1^{PS} F_{1,M_h,M_c}^{PS} \frac{(R_1^{PS}-1)}{R_1^{PS}} \frac{(\hat{T}_h^{in}-T_h^{PS,1})}{\ln\left(\frac{1-P_1^{PS}}{1-P_1^{PS} R_1^{PS}}\right)} R_1^{PS} \neq 1 \quad (\text{B-25})$$

$$Q_i^{PS} = U_{i,al,M_h,M_c}^{PS} A_i^{PS} F_{i,M_h,M_c}^{PS} \frac{(R_i^{PS}-1)}{R_i^{PS}} \frac{(\hat{T}_h^{in}-T_h^{PS,i})}{\ln\left(\frac{1-P_i^{PS}}{1-P_i^{PS} R_i^{PS}}\right)} R_i^{PS} \neq 1 \quad (\text{B-26})$$

$$Q_N^{PS} = U_{N,al,M_h,M_c}^{PS} A_N^{PS} F_{N,M_h,M_c}^{PS} \frac{(R_N^{PS}-1)}{R_N^{PS}} \frac{(\hat{T}_h^{in}-T_h^{PS,N})}{\ln\left(\frac{1-P_N^{PS}}{1-P_N^{PS} R_N^{PS}}\right)} R_N^{PS} \neq 1 \quad (\text{B-27})$$

Again, adding all these expressions, and using the fact that  $R_i^{\mathcal{P}\mathcal{S}} = N \hat{R}_T$  and  $A_T^{\mathcal{P}\mathcal{S}} = NA_i^{\mathcal{P}\mathcal{S}}$ , renders

$$\hat{Q}_T = \sum_{i=1..N} Q_i^{\mathcal{P}\mathcal{S}} = U_{i,al,M_h,M_c}^{\mathcal{P}\mathcal{S}} \frac{A_T^{\mathcal{P}\mathcal{S}}}{N} F_{i,al,M_h,M_c}^{\mathcal{S}\mathcal{P}} \frac{(N\hat{R}_T-1)}{N\hat{R}_T} \frac{\sum_{i=1..N} (\hat{T}_h^{in} - T_h^{\mathcal{P}\mathcal{S},i})}{\ln\left(\frac{1-P_1^{\mathcal{P}\mathcal{S}}}{1-P_1^{\mathcal{P}\mathcal{S}}N\hat{R}_T}\right)} N\hat{R}_T \neq 1 \quad (\text{B-28})$$

Now, we write

$$\begin{aligned} \prod_i (1 - P_i^{\mathcal{P}\mathcal{S}}) &= (1 - P_i^{\mathcal{P}\mathcal{S}})^N = \frac{(\hat{T}_h^{in} - T_c^{\mathcal{P}\mathcal{S},1})}{(\hat{T}_h^{in} - \hat{T}_c^{in})} \dots \frac{(\hat{T}_h^{in} - T_c^{\mathcal{P}\mathcal{S},i})}{(\hat{T}_h^{in} - T_c^{\mathcal{P}\mathcal{S},(i-1)})} \dots \frac{(\hat{T}_h^{in} - \hat{T}_c^{out})}{(\hat{T}_h^{in} - T_c^{\mathcal{P}\mathcal{S},(N-1)})} = \\ &= \frac{(\hat{T}_h^{in} - \hat{T}_c^{out})}{(\hat{T}_h^{in} - \hat{T}_c^{out})} = 1 - \hat{P}_T \end{aligned} \quad (\text{B-29})$$

Therefore,

$$P_1^{\mathcal{P}\mathcal{S}} = 1 - (1 - \hat{P}_T)^{1/N} \quad (\text{B-30})$$

With this result, we have:

$$(1 - P_i^{\mathcal{P}\mathcal{S}} R_i^{\mathcal{P}\mathcal{S}}) = (1 - P_i^{\mathcal{P}\mathcal{S}} \hat{R}_T N) = 1 - [1 - (1 - \hat{P}_T)^{1/N}] \hat{R}_T N \quad (\text{B-31})$$

Thus:

$$\hat{Q}_T = U_{i,al,M_h,M_c}^{\mathcal{P}\mathcal{S}} A_T^{\mathcal{P}\mathcal{S}} F_{i,M_h,M_c}^{\mathcal{P}\mathcal{S}} \frac{(N\hat{R}_T-1)}{N\hat{R}_T} \frac{(\hat{T}_h^{in} - \hat{T}_h^{out})}{\ln\left(\frac{(1-\hat{P}_T)^{1/N}}{(1-\hat{R}_T N [1 - (1-\hat{P}_T)^{1/N}])}\right)} N\hat{R}_T \neq 1 \quad (\text{B-32})$$

which, when compared with the overall expression (B-20) renders

$$F_{T,al,M_h,M_c}^{\mathcal{P}\mathcal{S}} = F_{i,al,M_h,M_c}^{\mathcal{P}\mathcal{S}} \frac{(N\hat{R}_T-1)}{N(\hat{R}_T-1)} \frac{\ln\left(\frac{1-\hat{P}_T}{(1-\hat{P}_T)\hat{R}_T}\right)}{\ln\left(\frac{(1-\hat{P}_T)^{1/N}}{(1-\hat{R}_T N [1 - (1-\hat{P}_T)^{1/N}])}\right)} N\hat{R}_T \neq 1 \quad (\text{B-33})$$

We take the limit of this expression for  $\hat{R}_T \rightarrow 1/N$  (using the L'Hôpital rule), to obtain

$$F_{T,al,M_h,M_c}^{\mathcal{P}\mathcal{S}} = F_{i,al,M_h,M_c}^{\mathcal{P}\mathcal{S}} \frac{\ln\left(\frac{1-\hat{P}_T/N}{1-\hat{P}_T}\right)}{(N-1)\left\{\left(\frac{[1-(1-\hat{P}_T)^{1/N}]}{(1-\hat{P}_T)^{1/N}}\right)\right\}} N\hat{R}_T = 1 \quad (\text{B-34})$$

Gardner (1942) used the same assumptions to develop a relationship between the individual values of  $F_i$  and the global  $F$  for this type of configuration.